Comment on "Air bubbles in water: A strongly multiple scattering medium for acoustic waves" [Phys. Rev. Lett. 84, 6050, 2000]

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In this communication, we would like to comment on some results in a recent Letter by Kafesaki *et al.* [1].

In [1], the authors stated "Up to now the extensive studies of bubbly water (or other bubbly liquids) have been analyzed in the framework of single scattering". In fact, acoustic scattering by bubbles has also been studied in terms of multiple scattering both extensively and intensively in the literature. This includes both theoretical (e. g. [2–12]) and experimental works (e. g. [13,14]). The work of Foldy [15], Lax [16], Waterman *et al.* [17], and Twersky [18] serves as a solid foundation to the multiple scattering of acoustic waves by many scatterers in general. The textbook by Ishimaru [19] presents an excellent account of various theories about the multiple scattering. The authors further suggested "Thus, acoustic waves in bubbly liquids (especially water), in addition to being very important *per se*, provide an almost ideal case for examining in detail the localization question". We note that the research on multiple-scattering induced acoustic localization in random bubbly water is rich (e. g. [4,5,11,20–24]). It has been shown [11,23,24] that acoustic waves can be localized within a range of frequencies slightly above the natural frequency of an air-bubble in water, and the localization behavior is insensitive to the configuration of bubble clouds [22]. When waves are localized in bubbly water, a global coherent behavior appears. This is a distinct feature differentiating the localization effect from residual absorption [24,26,27]. The localization disappears when the air-bubble volume fraction is lower than about 10^{-5}, in agreement with an earlier prediction [4].

To the end of their Letter, Kafesaki [1] stated "The most significant and novel results we obtained are shown in the panel of Fig. 3(b)...". The results shown in their Fig. 3(b) are a natural extension of the results of [11]. The formulation in [11] stems from the work of Foldy [15] and Twersky [18] and has been documented in detail in [25,28]. The approach has also been compared favorably with others in certain situations [25,28–30], and the results agree with the physical intuition (e. g. [31]). Though not exact, the approach is applicable for most frequencies considered in [1]. There will be a small inaccuracy when $\omega r_s/c$ exceeds 0.45 and significant inaccuracy is expected when $\omega r_s/c > 1$.

Using the formulations in [11,28] for acoustic transmission and using the formulation in [32] for computing the acoustic band structure, we examine the results in Fig. 3 of [1]. It should be noted that the formulation in [11] only takes into account the pulsating mode, while the formulation in [28] incorporates all possible vibrational modes of the bubbles. The two approaches agree well except at higher resonance frequencies of $\omega r_s/c \approx 0.48$ and 0.77 [1]. Thus, in the following, the approach in [11] will be used.

Employing the same parameters as in [1], we reproduce partial results in Fig. 1. For comparison, we show the result for a single random configuration (a) and as well as the ensemble averaged result (b). It is clear that whether making ensemble average makes not much difference for $\omega r_s/c_0 < 0.6$. In particular, the ensemble average basically plays no role for frequencies located around the complete band gap. Compared to Fig. 3 in [1], there are several differences. (1) The complete band gap is a little wider in the present case, but is consistent with a transmission calculation, following [22]. To further inspect the difference, we have also computed the band structures at the air void fraction of 10%, yielding nearly the same answer as in Fig. 1(a) of [1]. (2) In contrast to [1], the transmission in the present computation changes sharply at the frequencies slightly above the natural frequency of an air-bubble in water [2], which in fact leads to a first order phase transition from the extended state to the localized state [22]. Exploring the difference, we have considered possible effects of the locations of transmitting source and receiver on the transmission, and find that the transmission is insensitive to the positions of the transmitter and the receiver for frequencies between $\omega r_s/c_0 = 0.0136$ and about 0.3, indicating that the discrepancy in the transmission behavior between the present results and that from [1] is not caused by a difference in the source or receiver location. The transmission at frequencies higher than roughly $\omega r_s/c_0 = 0.3$, however, is indeed sensitive to the location of either transmitter or receiver. (3) In Fig. 3 of [1], the range of inhibited transmission coincides with the band gap, while in our case this is not the case. In addition, our results show that the randomness tends to smear out the transmission peak near the upper band edge, consistent with 2D cases [27]. At first sight, the present inconsistency between the range of the band gap and the range of the inhibited transmission seems to suggest errors in our computation. But, in fact, the the disagreement is due to the effect of the finite sample size. To elaborate, Fig. 2 shows the results for different sizes of bubble clouds, where the effect of finite sample size is obvious. The range of the transmission inhibition shall agree with the band gap when the sample size tends infinity.

When the multiple scattering is switched off, the effect of ensemble average becomes more prominent,

comparing the dotted lines in the right panels in Fig. 1(a) and (b). Again we see discrepancies with Fig. 3 in [1]. For example, the present result for the lattice case starts to flat out at about $\omega r_s/c_0 = 0.3$, while that in [1] at about 0.4. The randomness tends to broad the transmission peak located around the natural frequency of the breathing mode of the air-bubble. The two transmission peaks at $\omega r_s/c_0 = 0.48$ and 0.77 shown in [1] are absent from the right panel of Fig. 1. This is because the results in Fig. 1 did not take into consideration higher vibrational modes. When the higher modes are considered using [28], the overall results in the right panel of Fig. 1 will not change much, except that two sharp transmission peaks will appear as in Fig. 3(c) of [1].

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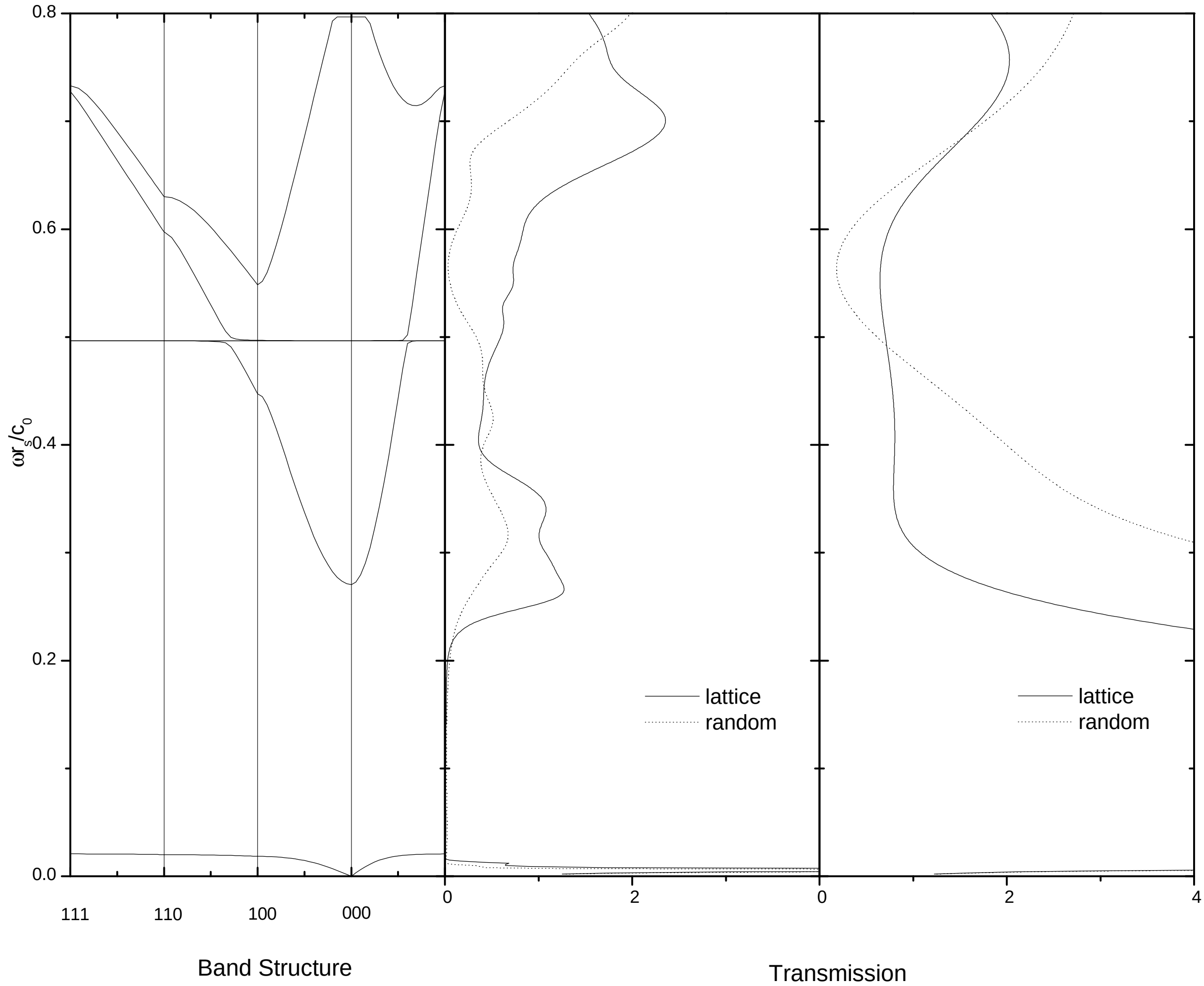
[1] M. Kafesaki, R. S. Penciu, and E. N. Economou, Phys. Rev. Lett. **84**, 6050 (2000).
[2] P. M. Morse and H. Feshbach, *Methods of theoretical physics*, McGraw-Hill, New York (1953).
[3] K. W. Commander and A. Prosperetti, J. Acoust. Soc. Am. **85**, 732 (1989).
[4] D. Sornette and B. Souillard, Europhys. Lett. **7**, 269 (1988)
[5] C. A. Condat, J. Acoust. Soc. Am. **83**, 441 (1988).
[6] A. S. Sangani, J. Fluid Mech. **232**, 221 (1991).
[7] Z. Ye and L. Ding, J. Acoust. Soc. Am. **98**, 1629 (1995).
[8] C. Feuillade, J. Acoust. Soc. Am. **99**, 3412 (1996).
[9] Z. Ye, J. Acoust. Soc. Am. **102**, 1239 (1997).
[10] F. S. Henyey, J. Acoust. Soc. **105**, 2149 (1999).
[11] Z. Ye and A. Alvarez, Phys. Rev. Lett. **80**, 3503 (1998).
[12] S. Temkin, J. Acoust. Soc. Am. **108**, 126 (2000).
[13] E. Silberman, J. Acoust. Soc. Am. **29**, 925 (1957).
[14] M. Nicholas, R. A. Roy, L. A. Crum, H. Oguz, and A. Prosperetti, J. Acoust. Soc. Am. **95**, 3171 (1994).
[15] L. L. Foldy, Phys. Rev. **67**, 107 (1945).
[16] M. Lax, Rev. Mod. Phys. **23**, 287 (1951).
[17] P. C. Waterman and R. Truell, J. Math. Phys. **2**, 512 (1961).
[18] V. Twersky, J. Math. Phys. **3**, 700 (1962).
[19] A. Ishimaru, *Wave propagation and scattering in random media*, Academic Press, New York (1978).
[20] D. Weston, J. Acoust. Soc. Am. **39**, 316 (1966).
[21] I. Tolstoy and A. Tolstoy, J. Acoust. Soc. Am. **87**, 1038 (1990).
[22] Z. Ye and H. Hsu, cond-mat/0006352; H. Hsu, *Localization of acoustic waves in bubbly water*, M. Sc. thesis, 87NCU00198017, NCU (1999).
[23] A. Alvarez and Z. Ye, Phys. Lett. **A 252**, 53 (1999).
[24] Z. Ye, H. Hsu, E. Hoskinson, and A. Alvarez, Chin. J. Phys. **37**, 343 (1999).
[25] Z. Ye and C. Feuillade, J. Acoust. Soc. Am. **102**, 798 (1997).
[26] E. Hoskinson and Z. Ye, Phys. Rev. Lett. **83**, 2734 (1999).
[27] e. g. Z. Ye and E. Hoskinson, cond-mat/0005183.
[28] A. Alvarez, C. C. Wang, and Z. Ye, J. Comp. Phys. **154**, 231 (1999).
[29] Z. Ye and A. Alvarez, J. Comp. Acoust. (*in press*)
[30] G. C. Gaunaurd and H. Huang, IEEE J. Ocean. Eng. **20**, 285 (1995).
[31] M. Strasburg, IEEE J. Ocean Eng. **21**, 233 (1996).
[32] M. S. Kushwaha, Int. J. Mod. Phys. **B 10**, 977 (1996); and reference therein.

Figure 1 (a) **Left panel**. Acoustic bands in a simple cubic array of air-bubbles in water. r_s is the bubble radius, ω is the angular frequency and c_0 is the water velocity in the water. Air volume fraction is 1%. **Middle panel**. Reduced transmission coefficient through the bubble array in ordered (solid line) or one random (dotted line) placement. **Right panel**. The same as in the middle panel, but with the multiple scattering switched off. (b) The same as in (a), but the transmission in the case of random placement is averaged over 100 random configurations.

Figure 2 The same as in Fig. 1, but for various sizes of bubble clouds in ordered placement (a) and as well as for one random arrangement (b).

Fig.1(a)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Band Structure
Transmission

Fig.1(b)

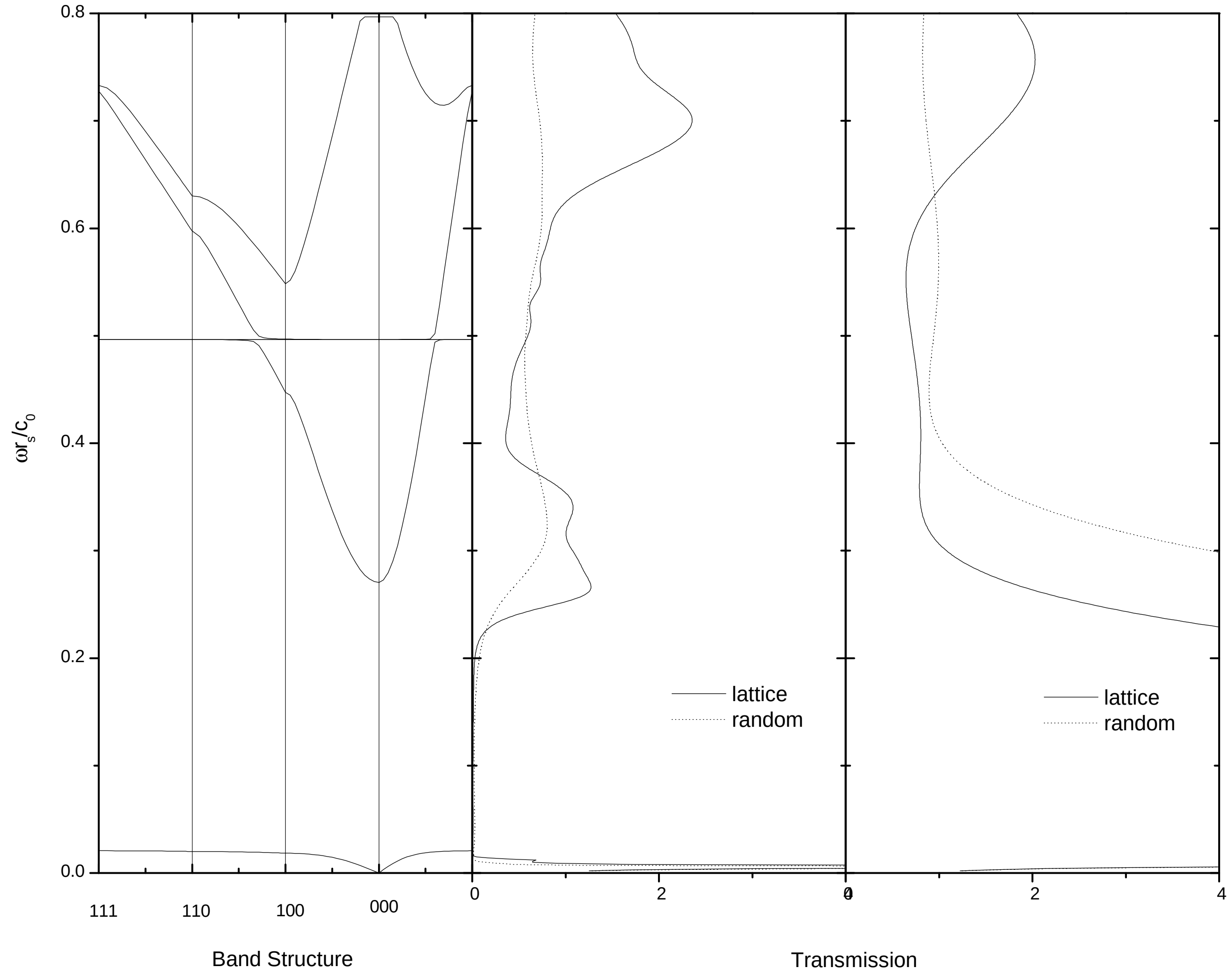

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Fig.2(a)

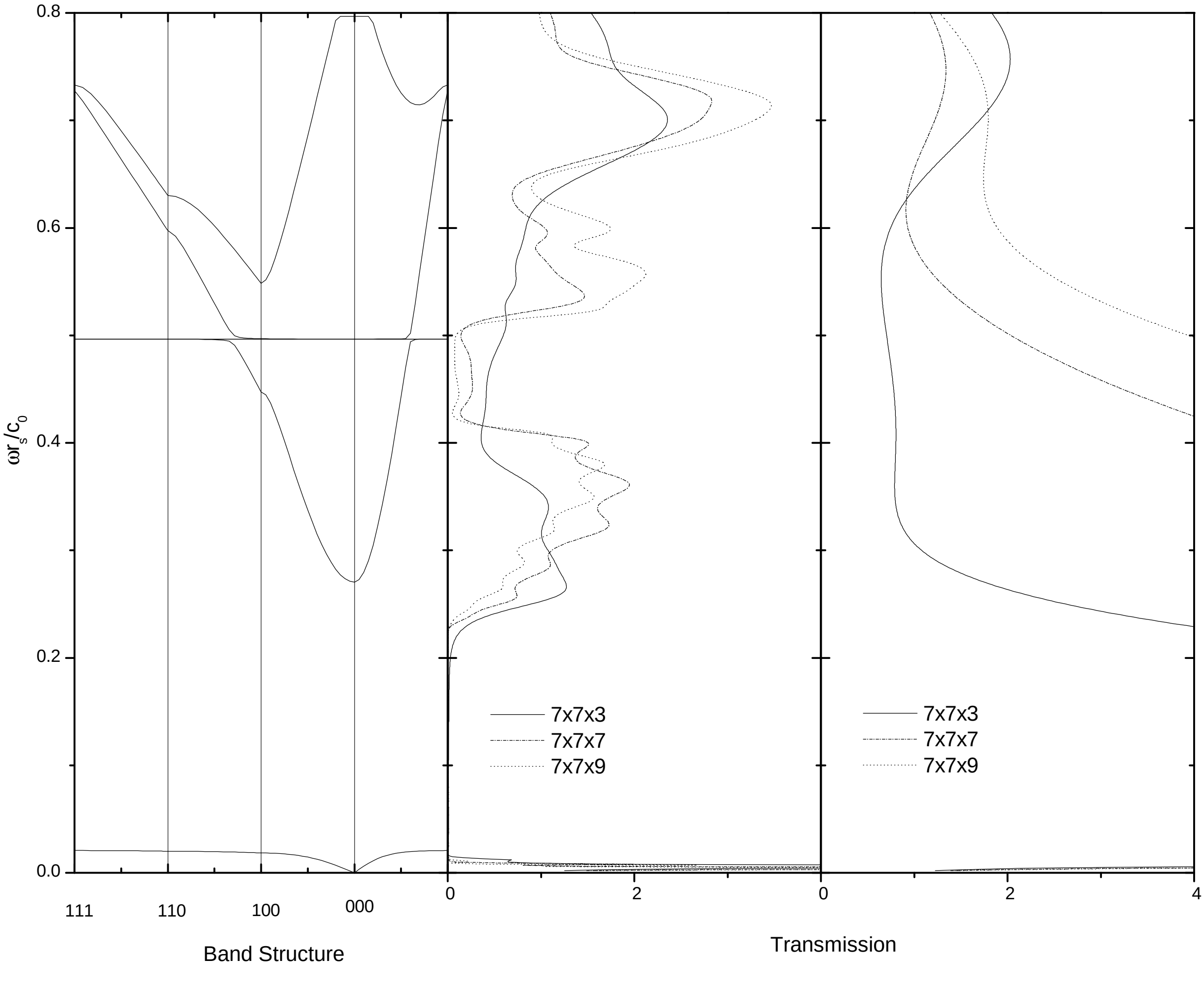

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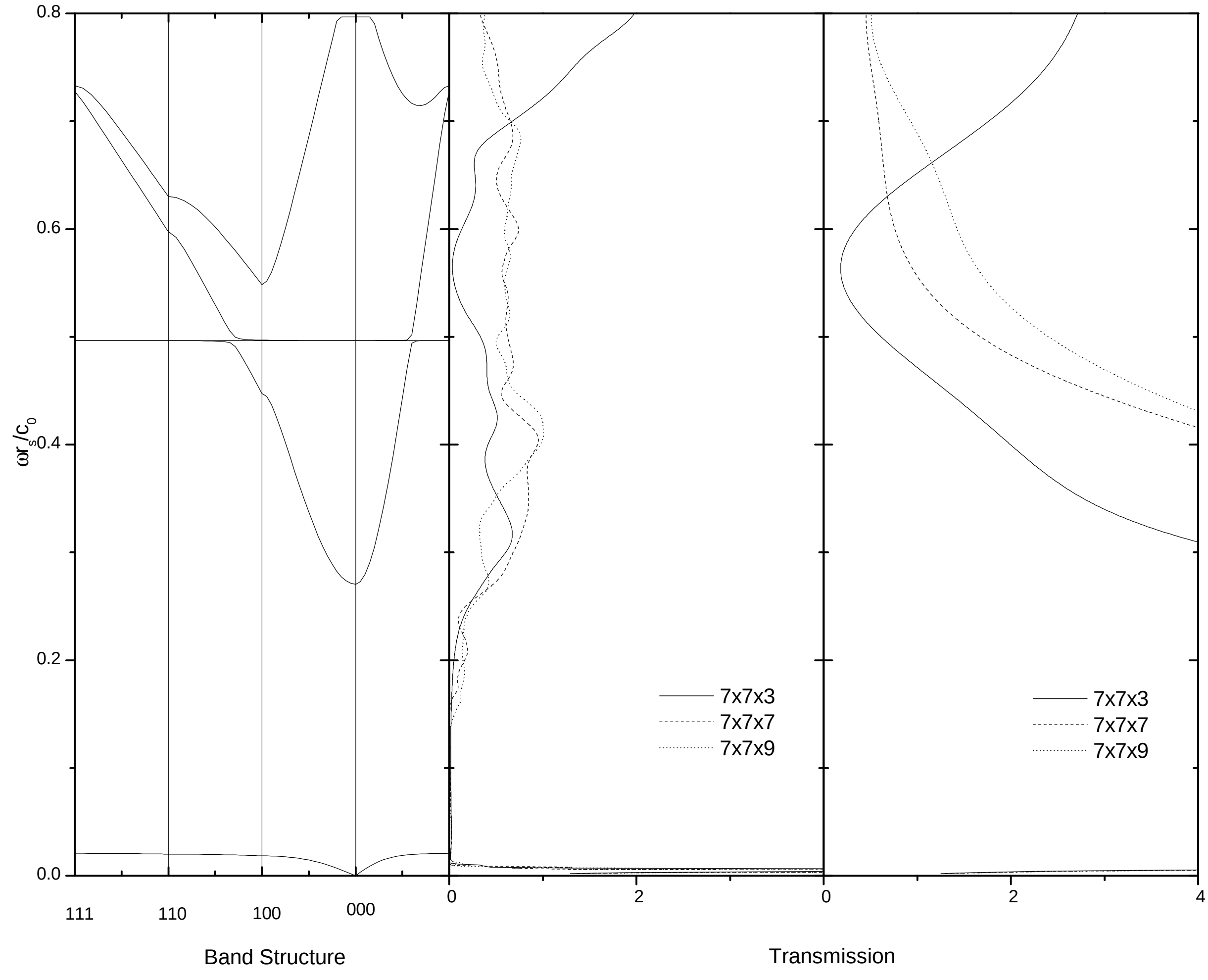
Fig.2(b)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